53479 Tidal Misconceptions

http://www.lhup.edu/~dsimanek/scenario/tides.htm

by Donald E. Simanek

The famous twin lighthouses at Folly Beach S.C.*

The word "tide" has two different meanings.

1. The variation of sea level at a coastal location, which depends strongly on shoreline topography, and on ocean currents near shore.
2. The deformation of land and water of the earth due to the gravitational forces of the moon and sun acting on every part of the earth.

It is the second meaning that is much abused in textbooks, and we will focus our atention on the deformations of the figure (shape) of the earth caused by the gravitational fields of the moon and sun. We will confine our discussion to the effect of the moon, which raises the largest tides, and illustrates the mechanism common to both lunar and solar tides.

Confusions begin when a textbook discussion of tides fails to define the word "tide", apparently assuming everyone knows its meaning. One of the few books that clearly defines "tide" at the outset is **The Planetary System** by Morrison and Owen [1966]: **"A tide is a distortion in the shape of one body induced by the gravitational pull of another nearby object."** This is definition (2) above. It clearly says that tides are the result of gravitation, without any mention of rotation effects.

How to distinguish tidal effects from other earth shape distortions. Lunar tides have a periodic variation tied to the periodic cycle of the moon's position in the sky (about 24 hours 50 minutes). The smaller solar tides are linked to the periodic cycle of the sun's position in the sky (24 hours). In fact, tides also occur at periods half this large (semidiurnal tides), and we wish to examine why.

Another thing that causes distortion of the shape of the earth is its axial rotation. Rotation changes the stress on water and land due to acceleration of these materials as they move in a circular path. This is responsible for the so-called "equatorial bulge" due to the earth's axial rotation. This raises the equator some 23 kilometers (0.4%) above where it would be if the earth didn't rotate. This is **not** a "tidal" effect, for it isn't due to gravitational fields of an external mass, and it has no significant periodic variations. This oblate shape is the reference baseline against which real tidal effects are measured.

Common misleading textbook treatments of tides.

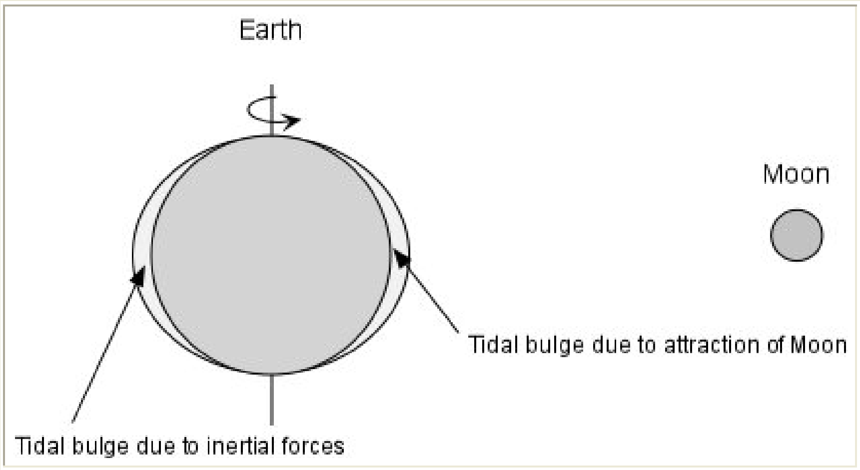
First, let's look at those textbook and web site treatments that generate misconceptions. Some of them, we strongly suspect, are the result of their author's misconceptions.

The subject of tides is complex, perhaps too complex to treat fully and satisfactorily in a freshman-level textbook. For this reason, many such books wisely ignore the subject entirely. Even some advanced undergraduate level mechanics texts dismiss the subject with a few sentences and the disclaimer "Consideration of the details would lead us too far astray." That's prudent.

But one question is certain to come up if even a *description* of tides is given in a class. "Why is there a high tide when the moon is overhead, and another high tide about 12 1/2 hours later?" That is, "Why is there a tide on the side of the earth nearest the moon, and also a tide on the opposite side of the earth from the moon?" Certainly that is an important question, one that any curious person would like to see answered.

> Since the moon advances in its orbit each day, the time between successive crossings of the observer's meridian is 24 hours 50 minutes. So the time between the two tidal bulges is half of that, or 12 hours 25 minutes.

The "two tides, two reasons" fallacy.

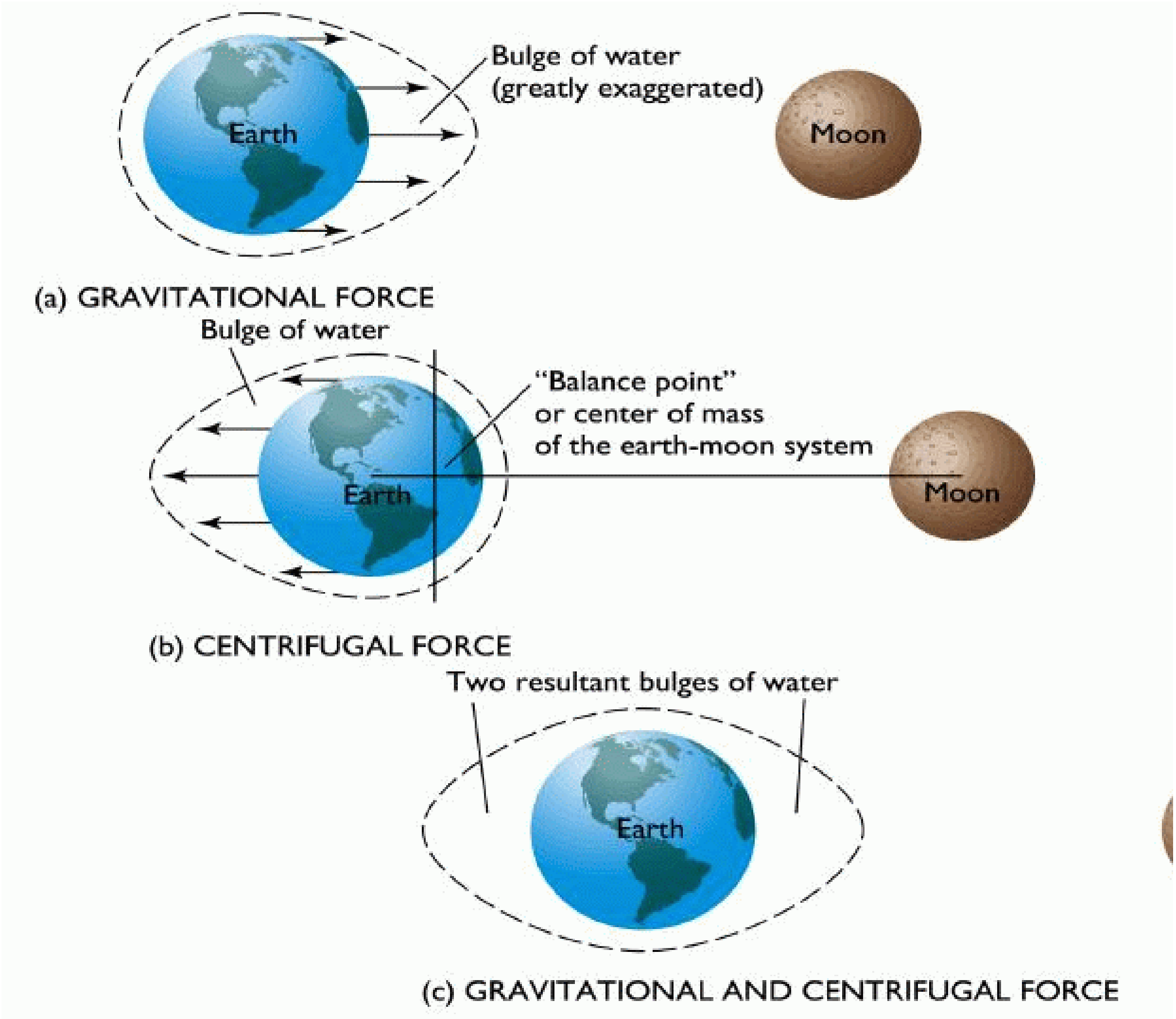


Misleading example seen in some textbooks.

Any student looking at this textbook illustration would conclude that the tidal bulge nearest the moon is entirely due to gravitation, while the bulge opposite the moon is due to "inertial effects". Sounds neat, and the diagram looks impressive, but it just doesn't stand up to analysis.

The diagram below compounds this error by breaking the diagram into three diagrams, and adding even more mistakes. The top figure shows a supposed single tide due to the moon's gravitational attraction. The second figure (below) shows a single tide "due to rotation of the earth" about a "balance point" that is the center of mass of the earth-moon system (the barycenter). What are those arrows shown in the figures? Context suggests that they are force vectors—centrifugal forces. Centrifugal force is a concept that is only applicable to solution of problems in rotating (non-inertial) coordinate systems. The accompanying text does not say whether the earth is assumed to be rotating with respect to the moon. It doesn't say whether the analysis is done in a rotating coordinate system.

We will see later that even when a rotating coordinate is assumed, one in which the moon is stationary, the size of the centrifugal force is the same size anywhere on or within the earth. The figure shows the arrows as clearly of different sizes, larger at points farthest from the barycenter. So what can they possibly mean?

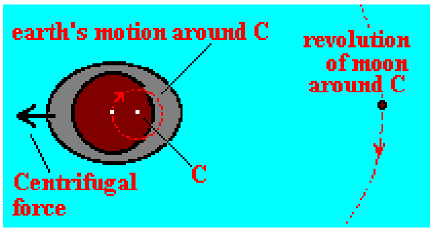


Now it could be that the arrows are only meant to suggest the **displacements** of water. If so, the caption should have said so. This diagram has many elements that can lead to misinterpretation, and strongly suggests the author or artist also had such misconceptions.

At this point, we strongly urge you to read, or at least review, a document explaining centripetal force.

Why can't they be consistent?

This curious example shows the earth-moon system as seen looking up toward the Southern hemisphere of the earth, or else it has the moon going the "wrong way". The accompanying text with this picture was no help at all. The (almost) universal textbook convention is to show these pictures as seen looking down on the Northern hemisphere of the earth, in which case the earth rotates counter-clockwise, and the moon orbits counterclockwise as well. It's getting so you can't trust pretty diagrams from any internet or textbook source.



Many textbook pictures show the moon abnormally close to the earth. Therefore the arrows representing the moon's gravitational forces on the earth are clearly non-parallel. But in the actual situation, drawn to scale, the moon is so far away relative to the size of the earth that those arrows in the diagram would be indistinguishable from parallel.

Misconceptions lead to false conclusions

These pictures, and their accompanying discussions, would lead a student to think that tides are somehow dependent on the rotation of the earth-moon system, and that this rotation is the "cause" of the tides. We shall argue that the "tidal bulges" which are the focus of attention in many textbooks, are in fact not due to rotation, but are simply due to the gravitational field of the moon, and the fact that this field has varying direction and strength over the volume of the earth.

These bulges distort the shape of the solid earth, and also distort the oceans. If the oceans covered the entire earth uniformly, this would almost be the end of the story. But there are land masses, and ocean basins in which the water is mostly confined as the earth rotates. This is where rotation does come into play, but not because of inertial effects, as textbooks would have you think. Without continents, the water in the ocean would lag behind the rotation of the earth, due to frictional effects. But with continents the water is forced to move with them. However, the frictional drag is still important. Water in ocean basins is forced to "slosh around", reflecting from continental shelves, setting up ocean currents and standing waves that cause water level variations to be superimposed on the tidal bulges, and in many places, these are of greater amplitude than the tidal bulge variations.

What's missing?

Too often textbooks try to toss off the tides question with a superficial analysis that ignores some things that are absolutely essential for a proper understanding. These include:

- Failure to define the specific meaning of "tide".
- Failure to properly define and properly use the terms "centripetal" and "centrifugal".
- Failure to say whether the analysis is being done in a non-inertial rotating system.
- Failure to warn the student that the force diagrams are different depending on whether the plane of the diagram is parallel to, or perpendicular to, the plane of the moon's orbit. If continents are shown on the earth, that's a clue. If part of the orbit of the moon is shown, that tells you diagram is in its orbital plane. But do students always notice these details?
- Neglect of tensile properties of solid and liquid materials.
- Neglecting to mention that liquid under stress physically moves toward a lower-stress configuration.
- Failure to specify the baseline earth shape against which a tide height is measured.

They are trying to get by "on the cheap".

So why are there tidal bulges on opposite sides of earth?

For a while we will set aside the complications of the actual earth, with continents, and look a the simpler case of an initially spherical earth entirely covered with water. If this rotates on its axis there's equatorial bulge of both earth and water, but we will treat this as a "baseline" shape upon which tidal bulges are superimposed.

The distortion of water and earth that we call "tidal bulges" is the result of deformation of earth and water materials at different places on earth in response to the combined gravitational effects of moon and sun. It is not simply the **size** of the force of attraction of these bodies at a certain point on earth that determines this. It is the **variation** of force over the volumes of materials (water and earth) of which the earth is composed. Some books call this variation the *differential force* or *tide-generating force (TGF)*.

The stress-producing effects of a non-uniform gravitational field acting on an elastic body are called **tidal forces**. Tidal forces

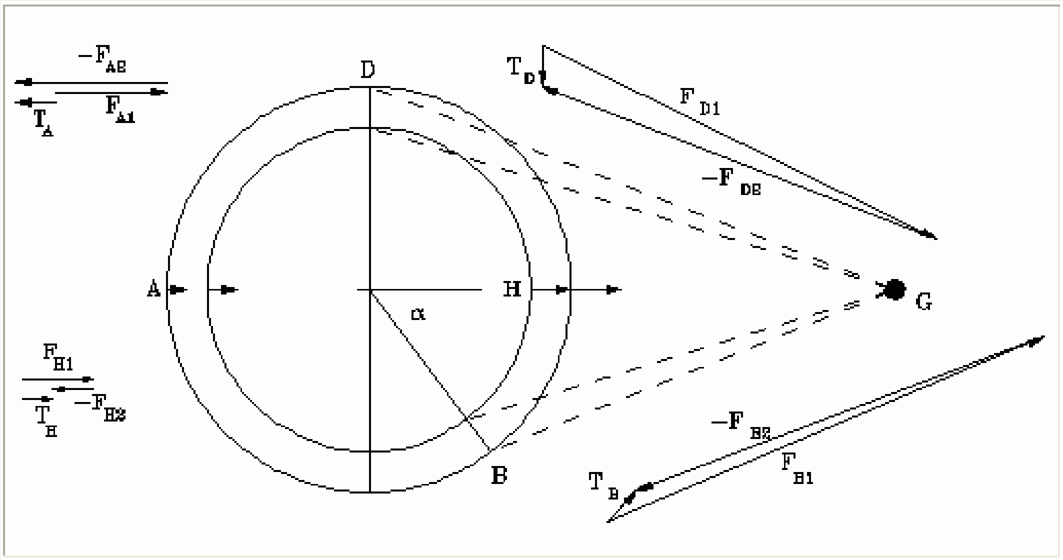
To find the distortion on a volume of the earth's material body we must sum the gravitational field of moon on that volume and the gravitational field of rest of the earth on that same volume, and then do the calculus operation of differentiation with respect to length.

from a gravitating body have a strength that depends in the inverse cube of the distance from that body, $F \sim 1/r^3$. Tidal forces are vector quantities, and may be drawn as arrows in a diagram, but the interpretation of such a diagram is different from that of a diagram of the gravitational forces themselves. Therefore textbooks should always specify which is being depicted.

Here's a short answer that attempts to get this idea across without explicit use of calculus.

Consider yourself standing on the earth's surface. The earth exerts downward forces on each part of your body, and the ground exerts an upward force on your feet. The net (sum) of these forces on your body is zero, and you are in equilibrium with respect to the ground. This is what keeps your feet on the ground, and also what holds the earth together in one nearly spherical piece. But these forces also cause compressional stress on your body. Each piece of your body supports everything above it, and experiences an upward force from whatever is just below it. This is the reference condition of stress at the surface of the earth, and we will see how this stress gets modified if another gravitating body exerts forces on a body, forces that vary in strength with distance.

We begin by considering a solid earth. While most discussions begin by considering the oceans, we will see that liquids and solids behave differently, so we begin with a solid, which is simpler. First look at a thick shell of solid material at the surface. We show this in the diagram, with the shell thickness exaggerated for clarity.



The tidal effects of an external gravitational field.
Differences in size of forces has been exaggerated.

The vector polygons do not have a uniform scale. </TH< tr>

We place a gravitating body **G** in the picture. All we need to assume now is that G exerts gravitational forces that decrease with distance and are all vectors directed toward G..

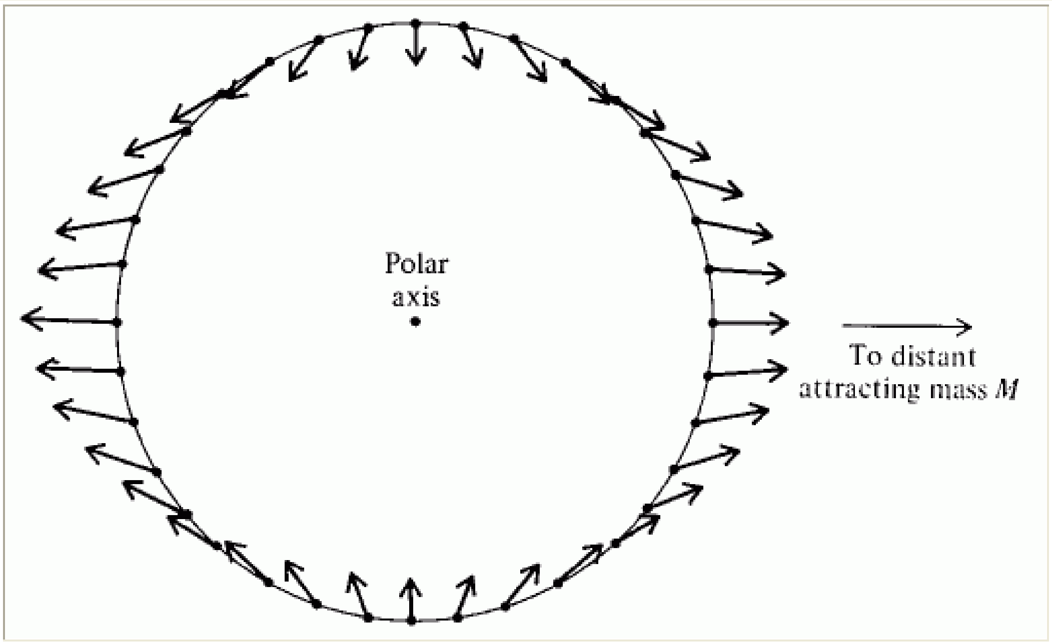
At point **H** on the surface of the earth the gravitational force from **G** is greater at the outer surface of the shell than at the inner surface. The stress there is $\boldsymbol{T_H = F_{H1} - F_{H2}}$. We adopt the convention that subscript 1 indicates the outer surface of the shell and subscript 2 indicates the inner surface. $\boldsymbol{T_H}$ is the tidal force, directed toward G. The vector diagram at lower left shows this.

At point **A** on the surface of the earth the gravitational force from **G** is greater at the inner surface of the shell than at the outer surface. The stress there is $\boldsymbol{T_A = F_{A1} - F_{A2}}$. $\boldsymbol{T_A}$ is the tidal force, this time directed away from **G**. The vector diagram at upper left shows this.

At point **D** the gravitational forces from **G** are nearly equal in size and nearly parallel. The vector diagram at upper right is long and skinny, and the tidal force here is pointing toward the center of the earth. This is a compression force.

At point **B** the tidal force is tangent to the earth's surface. There will always be such a point where there's no radial component of the tidal force, and for the real earth, it is at angle $\alpha = 54.7°$. The tangential component of the tidal force is sometimes called a *tractive* (pulling) force.

If this procedure is carried out for all places around the earth, a diagram of tidal forces can be constructed, which would look something like this:



Tidal forces, from Barger and Olsson.
The relative sizes of forces are exaggerated.

This diagram shows only the stress forces at the surface, but stress forces are distributed throughout the entire volume of the earth. One can now easily visualize how these shape-distorting stresses produce tidal bulges at both **A** and **H**. The deformation reaches equilibrium when the internal elastic forces in the solid body of the earth become exactly equal to the tidal forces.

At about 54.7° from the earth-moon line, the vector difference in the forces happens to be parallel to the surface of the earth. There the tidal force is directed horizontally. At this point there's no radial component of tidal force to produce compression stress, and the radius of the earth there is nearly the same as the radius of the unstressed earth. The horizontal components of tidal force push material toward the highest part of the tidal bulges.

The above description is appropriate for solid elastic materials. But if this globe were made entirely of liquid, the situation gets more interesting. Fluids move when forces are applied. They strongly resist compression or expansion. So if the water surface rises at high tide, water must have moved into the high-tide regions below and opposite the moon, coming from other portions of the ocean. This should not be surprising, for we know that water moves from higher to lower pressure regions in all situations, moving toward a configuration of equilibrium at lowest possible potential energy. For a liquid body, the tractive forces dominate, but the end result is still two tidal bulges when equilibrium is achieved.

How does this apply to the real earth?

...petroleum engineers who monitor pressure in large underground reservoirs of petroleum can watch an effect of these earth tides caused by the moon. The liquid-filled cavity in the rock below them is stretched and squeezed as the tides deform the solid earth, and the pressure rises and falls on their gauges twice each day.—Jay Bolemon [1985]

In the real earth, we have a solid crust with thin layers of ocean bounded by continents. The solid earth tides are dominated by the compressive-expansive radial components of the tidal forces. The large oceans are dominated by the tractive tangential components of the tidal forces. The interior of the earth behaves, in this context, like a solid elastic body, for mass movement of even the plastic materials cannot occur quickly enough. In either case, at equilibrium, the gravitational forces on each portion of matter are balanced by internal tensile forces.

The tidal bulges are very small, seemingly insignificantly small, compared to the radius of the earth. But over the huge area of one of the oceans, the tidal bulges alone still raise a huge amount of water. We have discussed these using the conceptual model of a stationary earth-moon system without continents, but with a uniform depth ocean covering its entire surface. We do this to emphasize that these tidal bulges are **not** due to rotation, but simply to the variation of the moon's gravitational field over the volume of the earth.

When we add continents to this model, the ocean bulges reflect from shorelines, setting up currents, resonant motions and standing waves. Also, the coastal topography (sea-floor slope and mouths of rivers and bays) can intensify water height fluctuations (with respect to the solid land). In fact, these effects are usually of greater size than the tidal bulges would be in a stationary earth-moon system. But most important is the fact that the whole complicated system, including the coastal tides, are driven by the tidal bulges discussed above, caused by the moon and sun. It is a tribute to the insight of Isaac Newton, who first cut through the superficial appearances and complications of this messy physical system to see the underlying regularities that drive it.

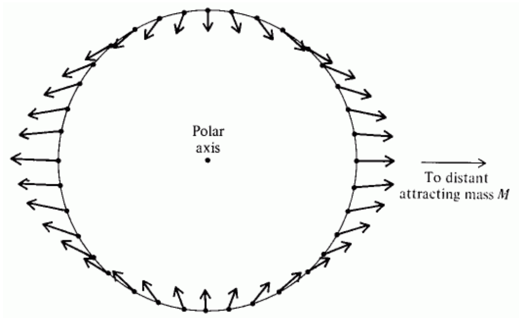
And even when we look at this more realistic model, recognizing the importance of "rotation", it is the rotation of continents (and their coastal geometry) with respect to water that gives rise to the complicated water level variations over the seas. It is not some mysterious effect of "centrifugal force" or "inertial effects" as some textbooks would mislead you to think.

We have ignored the stress due to the gradient of the earth's own field, because it is nearly the same amount anywhere on earth. We have also ignored the equatorial bulge of the earth, for we are treating that situation as the baseline against which the tidal effects are compared.

If all you wanted was the **reason** there are two tidal bulges, you needn't read further. I've even sketched out an even shorter treatment as a model for textbooks that have no need to go into messy details.

A picture of tidal forces.

Remember, when you see this diagram of tidal forces, it shows not the gravitational forces themselves, but the **differential** force, the **force gradient**, often called the **tide-generating force**. Similar pictures are found in other textbooks, with several different interpretations:



Tidal forces, from Barger and Olsson.

1. The picture shows simply the tide-generating forces due to the gravitational force of the moon. An inertial coordinate system is assumed, so there's no inclusion of centrifugal forces in the discussion.
2. The picture represents the vector sum of the moon's gravitational force and the centrifugal force in a rotating coordinate system, with the earth's gravitational forces not shown.
3. To avoid the messy details of talking about rotating coordinates and centrifugal forces, some books shortcut all this by loosely defining tidal forces as the difference between the actual lunar gravitational force at a point on earth and the lunar gravitational force at the center of the earth. Sometimes they call the latter the "average force" due to the moon. This produces a picture very like that above, but here the arrows represent real forces, not those "tidal forces". This interpretation is justified, if properly explained. These forces are proportional to the tensile force on columns of material extending from the center to the surface of the earth. To see how this approach works when done well, see Bolemon [1985].
4. The earth is accelerating in this inertial system, so the forces on any part of it do **not** sum to zero. Their sum, on any piece of mass, m, is m**a**, where **a** is the acceleration of that mass—a vector directed toward the moon.

Remember that in any of these interpretations, similar force summation is happening throughout the volume of the earth. Tidal forces stress the materials of the earth (earth and water), distorting the earth slightly into an ellipsoid. These diagrams are necessarily exaggerated, for if drawn to scale, the earth, even with tides and centrifugal effects would be a more perfect sphere than a well-made bowling ball (before the holes are drilled). Quincey has a good discussion of this, with diagrams. We can see from the diagram above how these combined forces distort the earth into an ellipsoid. But we can see from this photograph of earth from space, that all of the distortions due to rotation, and due to tides, are really very tiny relative to the size of the earth. Keep this photo in mind as you look at the drawings, which are necessarily greatly exaggerated.

The equilibrium theory of the tides.

Our simple analysis above also showed the importance of the relaxation of earth materials to achieve an equilibrium between gravitational forces and cohesive forces of materials. In more detailed analysis, we find that the figure (shape) of the earth at equilibrium is a constant shape consisting of two bulges nearly oriented in alignment with the moon. Underneath this equilibrium profile, the earth turns on its axis once a day, so the bulges move with respect to geography.

The fuller treatment of all of this is called the **equilibrium theory** of the tides. It is usually carried out in coordinate system, rotating about the barycenter of the earth-moon system. In this coordinate representation, the earth and the moon are considered stationary with respect to each other, and we ignore the daily rotation of the earth around its own axis.

An inertial system is one in which Newton's law, $\boldsymbol{F} = m\boldsymbol{a}$ holds for each part of the system, where $\boldsymbol{F}$ is the sum of all real forces on that part.

In this representation we can treat this system *as if* it were an inertial system, but only at the expense of introducing the concept of *centrifugal force*. It turns out that when this is done, the centrifugal force on a mass anywhere on or within the earth is of constant size, and is therefore equal to the size of the gravitational force the moon exerts on the same amount of mass at the center of the earth. We'll look at this in more detail below.

A closer look at centrifugal forces.

So what about those centrifugal forces so many books make such a fuss about? You'll notice we never mentioned them in our simple explanation. Should we have?

Many misleading accounts of the tides result from a common confusion about centrifugal effects due to rotation. Let's be very clear about this. The only forces that act on the body of the earth are:

- The gravitational forces between each part of the earth and every other part, and the gravitational forces on parts of the earth due to the moon, sun, and nearly negligible forces due to more distant bodies in the solar system.
- Internal tensile forces within the materials of the earth.

If a textbook mentions centrifugal forces without defining inertial systems and without telling the reader that this term has meaning only when using a rotating coordinate representation, you can reasonably suspect that the book may also be deceiving you in other ways. In brief, here's a review of details that can be found in any good intermediate-level undergraduate book on classical mechanics. The rotating coordinate method of dealing with this problem is preferred by professionals, but we remind you that its physical results are identical to those you'd get working out the problem in an inertial (non-accelerating) coordinate system.

Polar coordinates are most convenient for for problems such as this. The term "centripetal force" is nothing more than the radial component of the net force on a body. Such a body is accelerating, so it is not in equilibrium, and there's a net nonzero force on it. That is, the sum of all real forces on the body is **not** zero.

> Whether you use inertial or non-inertial coordinates for the analysis of a problem, the results (that you can observe in nature) must be the same.

When using the rotating non-inertial coordinate representation, it is customary to introduce "fictitious" forces called "centrifugal" forces. The centrifugal force on any part of the body is simply a force equal and opposite to the centripetal force there. This dodge has the handy result that when you add the fictitious forces to the real forces, the net force on the body is zero. Also, $\boldsymbol{F} = m\boldsymbol{a}$ holds, where $\boldsymbol{F}$ is the sum of real and fictitious forces. Then the body may be treated "as if" it were in equilibrium, and the familiar methods of solving equilibrium problems may be used. This trick has turned a problem about a non-inertial system into one that can be analyzed *as if it were an inertial system*.

Textbooks that introduce "centrifugal" language, but do **not** do any mathematical derivations, and do **not** explain or use rotating coordinate representations. are very likely to mislead the student into thinking that these "centrifugal" forces are some actual real forces, arising from some mysterious causes. These books may even equate these forces to "inertia", which doesn't help anyone understand anything. The very worst offenders even describe centripetal forces and centrifugal forces as "reaction" force pairs, as in Newton's third law. This makes no sense at all, for they have both of them acting on the **same** body. The action/reaction forces described by Newton's third law act on **different** bodies, by definition.

This figure, from French, explains the importance of centrifugal forces, which turns out to be of no importance at all unless you choose to do an analysis of the problem in a rotating coordinate system. As we saw above, you don't have to.

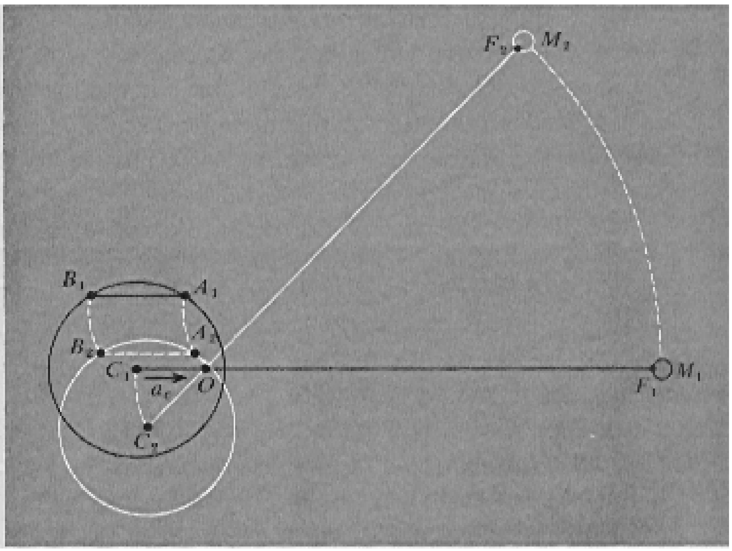


Fig. 12-27 The orbital motion of the earth about the moon does not by itself involve any rotation of the earth; the line A_1B_1 is carried into the parallel configuration A_2B_2.

We ignore the effects of the earth's rotation about its own axis, which of course underlies everything. The equatorial bulge it produces is the baseline against which tidal variations are referenced. We are now focusing on the effects due only to the earth-moon system. The motion of the earth about the earth-moon center of mass, causes every point on or within the earth to move in an arc of the same radius. This is a geometric result most books totally ignore, or fail to illustrate properly. Therefore every point on or within the earth experiences the same size centrifugal force. A force of constant size throughout a volume cannot give rise to tidal forces (as we explained above). The size of the centrifugal force is the same as the force the moon exerts at the earth-moon center of mass (the barycenter), where these two forces are in equilibrium. [This barycenter is 3000 miles from the earth center—within the earth's volume.]

So the bottom line is that centrifugal forces on the earth due to the presence of the moon are **not** tide-raising forces at all. They cannot be invoked as an "explanation" for any tide, on either side of the earth or anywhere else. So why do we find them used in "explanations" of tides in elementary-level books? Could it be because these text's authors are often misled by their own pretty diagrams? Once they launch into the rotating coordinate mode and start talking about centrifugal forces, they seem to forget that the earth's own gravitational field is still present and acting on every portion of matter on earth. They also forget that the non-uniformity of moon's gravitational field over the volume of the earth is alone sufficient to account for both tidal bulges, bulges that would be essentially the same if the earth-moon system were not moving, and not moving relative to each other.

Physicists call centrifugal forces "fictitious" forces, because they are only conceptual/mathematical aid to the analysis of rotating systems that **we choose** to analyze in a non-inertial coordinate system. [We didn't have to do it that way.] In such a system fictitious interpretations may arise, such as the notion that the tidal bulge opposite the moon is due entirely to inertial" (read "fictitious") forces, and the implication that gravitation has nothing to do with that bulge.

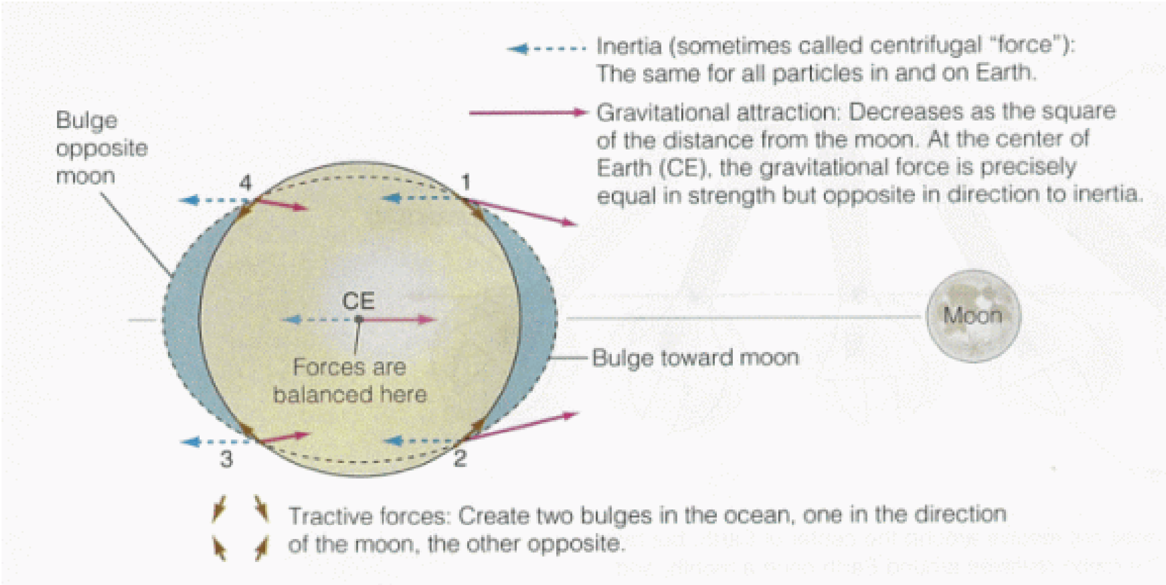
It must also be understood that these textbook pictures are static diagrams, "snapshots" of a dynamic system. The daily rotation of the earth underneath these "tidal bulges" causes the bulges to move around the earth each day. And all of these deformations sit "on top" of the equatorial bulge that goes all the way around the earth, due to the earth's daily rotation.

Coastal tidal variations

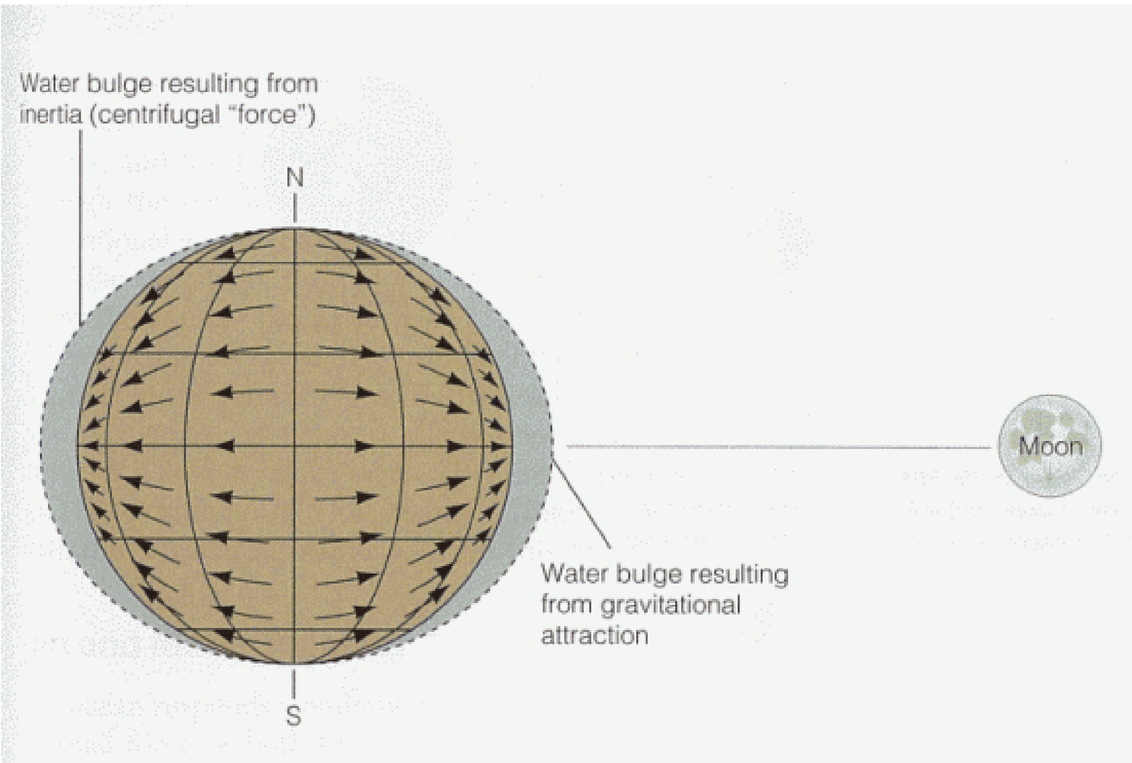
Finally, the coastal tides have considerable local variations because of differences of shoreline slope, and ocean currents. The oceans don't cover the entire earth, but "slosh around" daily within the confines of their shores. Reflections from shores can set up interference patterns farther out in the ocean. But the driving force for all of thess complications is still those two "daily" lunar tides (12 hours 25 minutes apart), which we have explained above, combined with the two much smaller daily tides (12 hours apart) due to the Gravitational field of the sun.

More misleading textbook illustrations.

An oceanography textbook has this diagram that at least shows centrifugal forces of equal size.



One is tempted to think "This book has it right!" But reading the text makes one suspicious. Then on the very next page we see this diagram in which the author identifies one tide as being from gravitation, the other from inertia.

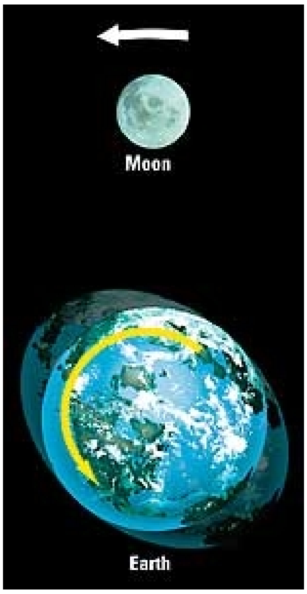


Comparing the two pictures, one sees that they contradict. The one that shows forces clearly suggests that the moon's gravitational force is responsible for both tides.

Unfortunately, like so many other books, this book fails to tell the student the origin of these centrifugal forces, and fails to emphasize that they are not "real" forces, but only a useful device to do problems in rotating coordinate systems.

Here the chickens come home to roost, for misunderstanding of centrifugal effects originates in some elementary-level physics textbooks. Nowhere does this book even suggest that rotating coordinate systems are being assumed.

Other lunar misconceptions.



Friction. We mentioned frictional drag of water on the ocean floor. The earth "drags" the tidal bulges. Similar drag effects are acting within the crust of the earth as well. This causes the tidal bulges to arrive a little "late" (compared to the time of the moon's crossing the observer's meridian). [The earth's rotation and the moon's revolution are both counter-clockwise as seen from above the N pole. The earth rotates faster than the moon revolves around the earth, so the earth drags the high tide bulge "ahead" of the moon. Therefore, as we move with respect to tidal bulge and moon, the moon crosses our meridian before the highest tide.] How early? Some books show misleading diagrams with the symmetry axis of the tidal bulges making an angle of 30° or more with the moon. In fact, the angle is only 3°, so the tides are late by about 24(3/360)60 = 12 minutes. We doubt that even the most avid surfer would consider this of great significance.

This has another important effect. The moon exerts a retarding torque on those tidal bulges. This is in a direction to gradually slow the earth's rotation. And the bulges exert an opposite torque on the moon, increasing its distance from earth, and reducing its velocity, as required by the law of conservation of angular momentum.

This exaggerated diagram, from a web site, shows an angle of about 45° between the bulges and the moon's position. But it does show correct rotation relationships. Such diagrams are often seen in textbooks. Of course, the relative sizes of earth and moon and their distance of separation are not to scale either. An accurate scale diagram wouldn't show any of these relatively small-scale phenomena. So exaggeration must be used to get the idea across. This wouldn't be so bad if the accompanying text clearly indicated that the distance and the angle have been exaggerated, but somehow that disclaimer is often forgotten. We have even seen some texts that don't anywhere indicate that the angle is only 3°.

Push-Pull language.

Often textbooks say something like this:

The moon pulls the ocean on the near side of the earth more than it pulls on the center of the earth. The pull on the ocean at the far side of the earth is smaller still. This causes the near ocean to accelerate toward the moon most, the center of the earth less, and the far ocean still less. The result is that the earth elongates slightly along the earth-moon line.

This conjures images of motion of parts of the earth moving continually toward the moon. But in the actual situation, the earth and moon remain a nearly constant distance apart; and this distance doesn't change appreciably during a lunar cycle.

This misleading "explanation" is often found in lower-level physics texts that try to use "colloquial" language to describe things too complex for such imprecise language. Some of these books even say, as if it were a definition: "A force is a push or a pull". To the student mind this implies motion. Oh, the textbooks do consider forces acting on non-moving objects, but the harm has already been done by the earlier statement that the student memorizes for exams.

This "differential pulling" language exists in textbooks in several forms. Sometimes the phrase "is pulled more" or even "falls toward the moon faster" is used. All begin with the assumption that earth and moon are in a state of **continually** falling toward each other, and that's a correct statement, though not likely to be clearly understood by students. But if this "falling" is continual, then the "pulling" refereed to in the example above is continual also. Now they bring in acceleration, and say that the lunar side of the earth accelerates most, the opposite side least. So, the student reasonably infers, the acceleration difference is continual.

Now if two bodies move in the same direction, the one with greater velocity will move more and more ahead of the other one. It's gain is even greater if the lead one has greater acceleration. If this "explanatory" language were to be believed as applying to the earth, the earth would continually stretch until it is torn apart.

This explanation goes astray because it doesn't acknowledge (1) the earth's own gravitational field and (2) tensile forces in the body of the earth. Also, they are using "force" language, without adhering to the fundamental principle of doing force problems: You must account for and include **all** forces acting on the body in question.

And, we suspect, the authors of these explanations may themselves have been misled by a misunderstanding of rotation and centripetal and centrifugal forces.

Some dirty little secrets textbooks fail to tell you.

The "tidal trivia" summary below puts things into perspective. The so-called equatorial bulge due to the earth's axial rotation lifts the equator about 23 kilometer. The moon's gravity gradient lifts water mid-ocean (where the ocean is deep) no more than 1 meter, that's 1.6×10^{-7}% of the earth radius. Why do we fuss about this? Because over an ocean of large area, that represents a very large volume of water. It's the driving mechanism that controls the periods of the much larger tides at shorelines.

The moon's gravitational forces act in two ways on the earth:

1. They stretch solid objects—an effect proportional to the inverse cube of the distance from the moon.
2. Their tangential components exert tractive forces on large bodies of water to move that water toward the high and low tides. These are also proportional to the inverse cube of distance from the moon. This is the dominant reason for tides in large bodies of water.

The reason water can rise as much as 1 meter in mid-ocean is primarily because the ocean is so large that water can move into the tidal bulge. The tidal rise in Lake Michigan is smaller because the lakes' area is much smaller. The tide in Lake Michigan would be about 2 inches [Sawicki]. Smaller still is the tide in your backyard swimming pool. It's unmeasurably small. Don't even bother with the tide in your bathtub or your morning cup of coffee. Oh, there's "stretching" tides in all of these, but the land, table and cup rise, and the coffee rises with it, by nearly the same amount, perhaps a fraction of a meter as the moon is high in the sky. But you don't notice anything unusual.

The concept of centrifugal force is handy for doing the calculations of the results of the last paragraph. But since all these effects are affecting only the "baseline" level of land and water, against which tidal variations are referenced, a discussion of tides does not need to mention centrifugal forces. That only invites confusion and misconceptions. Centrifugal forces are not tidal (tide-raising) forces.

The folks who do tidal measurements don't get into the physics theory much. Tide tables are constructed from past measurements and computer modeling that does not take underlying theory much into account. It is much like the pretty weather maps you see on TV, computer generated without a detailed understanding of all the physical details. The task is just too complicated for even our best computers, and the data fed into them is far from the quality and completeness we'd need.

You might think that with global positioning satellites we'd have the measurements of water and land tides accurate to a fraction of a smidgen. You'd be wrong. If you check the research papers of the folks who do this, you see that they are still dissatisfied with the reliability of such data even over small geographic regions. We can map the surface of land to within a meter this way, and get relative height measurements equally well, but absolute height measurements relative to the center of the earth are much poorer. Many of the numbers you see tossed about in elementary level books are copied from other elementary level books, without independent checking and without inquiring whether they were guestimates from theory or from actual measurement.

You may also think that modern computers have made tide prediction more accurate. In fact, the analog (mechanical) computers devised for this purpose in the 19th century did just as good a job, even if they have ended up in science museums.

Superposition

Astrophysicists also need to understand the physics of tides. They must deal with tides in a more general way, such as tidal forces acting on binary stars, and on rings of Saturn. Consider this clear description from Josh Colwell's web site, intended for his students.
If the tidal force can tear apart a strengthless fluid object (as in the derivation of the classical Roche limit), then it still applies some stretching force to solid, intact bodies, such as moons. First consider the static situation where neither the moon nor the planet are moving or rotating: the tidal force stretches each object resulting in bulges along the line connecting the two objects. These are called tidal bulges.

Now allow the moon to rotate. Because the bulge is produced by the differential force of gravity across the moon, it wants to stay aligned with the line connecting the two bodies. But if the moon is rotating, different parts of the physical body [of the moon] must go through this tidal bulge. The result is that as the moon rotates it is constantly being stretched and deformed. This of course takes energy, and that energy comes out of the rotation energy of the moon and results in heat energy (frictional dissipation of energy as the solid moon is deformed). As energy is removed from the moon's rotational energy its rotation slows to the point where it always keeps the same real estate pointed at the planet so it doesn't have to do any stretching or deforming. For it to keep the same real estate pointed at the planet as it goes around the planet, its rotation period must equal its orbit period. This is called synchronous rotation, and all major satellites in the solar system exhibit synchronous rotation.

This clear description uses a valuable conceptual approach to understanding tides—the principle of superposition. This principle is useful when several processes act together and the results of them are linearly additive. It allows us to consider each process separately and then combine the results. We first consider a separated earth and moon with no motion at all. They would have to be fastened in place somehow, but that's an unimportant detail. Such a static situation **will cause tidal bulges** on both bodies. This clearly tells us that such bulges are not due to motion, but are entirely due to gravitation.

The second paragraph looks at what happens to the moon when you add the rotation of the moon. But the same arguments can be extended, to see what happens to the earth as we add the rotation of the earth. The earth rotates much faster than the moon revolves, so the earth's tidal bulges "track" the moon, lying close to, but slightly ahead of, the line joining the earth and moon. They are "dragged" ahead by friction. The tidal bulges move across the earth's geography, **and** friction forces dissipate energy in the earth, slowing its rotation.

Then we can look at the gravitational torques acting on the tidal bulges. These act to slow the rotation of the earth also. But they also act to increase the moon's distance from earth and decrease its velocity. This is required by conservation of the total angular momentum of the earth-moon system. If the axial spin rotational momentum of the earth decreases (due to energy dissipation), then the orbital angular momentum of the moon must increase to compensate. Astronomy texts may be consulted for the very interesting long-term outcomes of these interactions.

Tidal trivia.

- Amplitude of gravitational tides in deep mid-ocean: about 1 meter.
- Shoreline tides can be more than 10 times as large as in mid-ocean.
- Amplitude of tides in the earth's crust: about 20 cm.
- The gravitational force of sun on earth is 178 times as large as the force of moon on earth.
- Ratio of sun/moon tidal forces on earth is 0.465.
- Tidal stretch of human body changes its height by fraction 10^{-16}, an amount 1000 times smaller than the diameter of an atom. By comparison, the stress produced by the body's own weight causes a fractional change in body height of 10^{-2}. [Sawicki]
- Tidal friction causes earth days to lengthen 1.6 milliseconds/century. [Sawicki]
- Angular velocity of earth's axial rotation: 7.29×10^{-5} rad/s.

- Angular velocity of moon's revolution around earth: 2.67 x 10^{-6} rad/s.
- earth polar diameter: 12710 km.
- earth equatorial diameter: 12756 km.
- Difference between these diameters: 46 km.
- Difference between these radii: 23 km, or 0.4 %.
- Centripetal acceleration at earth's surface due to earth's axial rotation: 0.034 m/s^2
- Size of centripetal acceleration at earth's surface due to earth's circular motion around the earth-moon barycenter: 3.3 x 10^{-5} m/s^2.

References

1. Abell, George O., Morrison and Wolff. **Exploration of the Universe.** Sixth Edition, Saunders, 1993. A short non-calculus treatment that is better than those found in most elementary Astronomy texts. Clearly the authors thought this through themselves; they didn't simply copy material found elsewhere. p. 68-70.
2. Arons, Arnold B. "Basic Physics of the semidiurnal lunar tide." **Am. J. Phys. 47** 11, Nov 1979. p. 934-937. Proposal for a noncalculus treatment without reference to fictitious forces and without recourse to a potential. Common misconceptions are noted.
3. Barger, Vernon D. and Martin G. Olsson. **Classical Mechanics, A Modern Perspective**. McGraw-Hill, 1973. Advanced undergraduate level, with clear discussion and illustration of tidal (tide-generating) forces. p. 265-274. This treatment does no use non-inertial coordinates.
4. Bolemon, Jay. **Physics, an Introduction.** Prentice-Hall, 1985. p. 129-134. An original, thorough, and correct, treatment at a non-mathematical level.
5. French, A. P. **Newtonian Mechanics**. Norton, 1965. An extensive discussion at an advanced undergraduate level. Of the books I've looked at this one has the best treatment of tides.
6. Morrison, David and Tobias Owen. **The Planetary System.** Addison-Wesley, 1996. p. 202. A brief and correct description in less than a page, which tells no lies.
7. Sawicki, Mikolaj. "Myths about Gravity and Tides". **The Physics Teacher, 37**, October 1999, pp. 438-441. A pdf revised version is available online. This article discusses a wide range of misconceptions about the tides.
8. Swartz, Clifford E. and Thomas Miner. **Teaching Introductory Physics, a Sourcebook**. American Institute of Physics, 1997. A brief discussion on pages

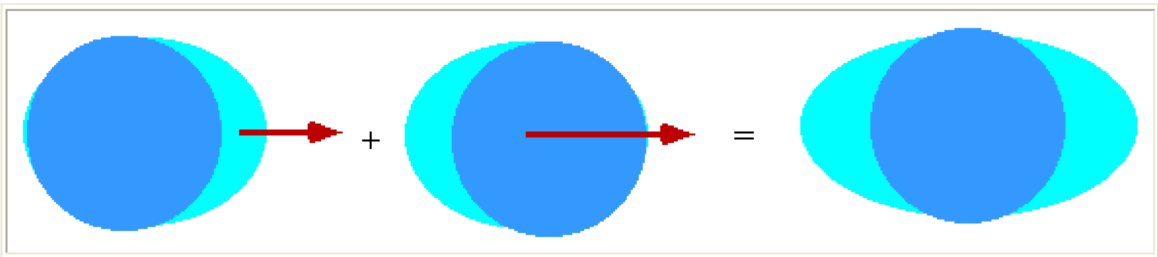
112-116.
9. Quincey, Paul. "Why We Are Unmoved As Oceans Ebb and Flow" in **Skeptical Inquirer**, Fall 1994, p. 509-515. Addresses misleading press and media misrepresentations of tides.
10. Tsantes, Emanuel. "Note on the Tides". **Am. J. Phys. 42** 330-333. A mathematical treatment that does not use rotating non-inertial coordinates and does not mention centrifugal force. Explicitly discusses role of elasticity of materials.

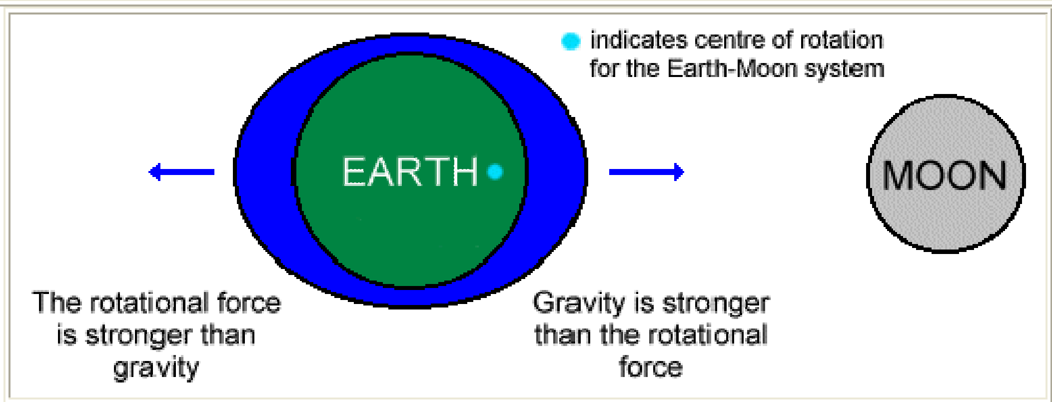
Web sites with reliable information.

- Basic Concepts in Physical Oceanography: Tides. Department of Oceanography, Naval Postgraduate School.
- Optenpedia discussion of tides, with references.
- NOAA. Our Restless Tides. A brief explanation of the basic astronomical factors which produce tides and tidal Currents.

Final Exam.

1. These pictures are from various internet sources. Find the misleading features of each.





2. If the earth were not rotating, and the moon stopped revolving around it, and they were "falling" toward each other, would the earth have tidal bulges? If not, why? If so, would they be significantly different from those we have now? In what way?

3. Here's an example of how untrustworthy textbooks are. This is from a college level introductory college physics text.

From this explanation (previously given) it would seem that the tides should be highest at a given location when the moon is directly overhead (or somewhat more than 12 hours later). In fact, high tide always occurs when the moon is near the horizon. The reason is that the friction of the rotating earth tends to hold the tides back so that they always occur several hours later than we should expect.

Find the **serious** error(s) in this short paragraph.

4. A web site has this gem of wisdom: "As the earth and moon whirl around this common center-of-mass [the earth/moon barycenter], the centrifugal force produced is always directed away from the center of revolution." Is there anything wrong with this statement?

5. [From Arons, 1979] If our moon were replaced by two moons half the mass of our moon, orbiting in the same orbit, but 180° apart, would the earth still have tides? If not, why not? If so, how would they compare with the tides we now have?

6. If the tides may be thought of as a "stretching" of the earth along the axis joining the earth and moon, then why are all materials not stretched equally, resulting in no ocean tides? If elastic strain is the reason for tides, then since the elastic modulus of water is so much smaller than rock, wouldn't you expect that rock would "stretch" more than water, causing water levels to drop when the moon is overhead? Explain.

7. When we say that the tide in deep mid-ocean is about half a meter, what is this measured with respect to? (a) a spherical earth, (b) an oblate earth with equatorial bulge, (c) the bottom of the ocean, (d) the ocean's shores (e) low tide.

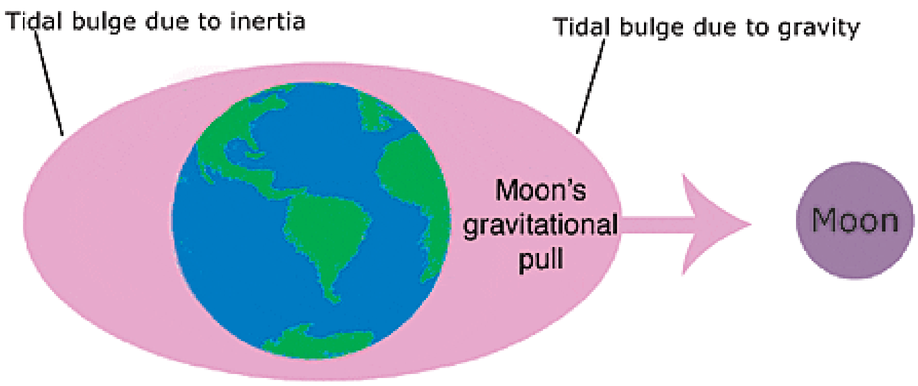
8. If the earth were in a rotating, uniform (parallel field lines, constant strength) external gravitational field (don't ask how we might achieve this), would we have tides at the period of its rotation? Would we have tides at the half-period of its rotation?

9. If a huge steel tank were filled with water, and a sensitive pressure gauge were put inside, would the pressure gauge register tidal fluctuations with a period of about 12.5 hours?

10. Textbooks sometimes say the tide on the side of earth opposite the moon is smaller than the tide on the side nearest the moon, because the moon's gravitational pull is weaker there, farthest from the moon. How much smaller, would you estimate

it to be at mid-ocean? The moon is about 60 earth radii away from the earth. How much smaller is the weaker high tide compared to the stronger one?

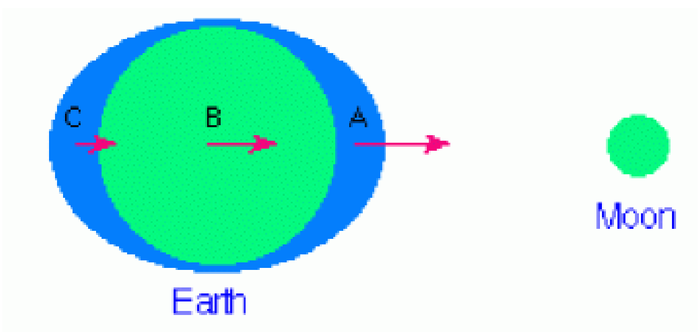
11. The picture and text below are from the NOAA-NOS website. Your tax dollars at work to propagate misconceptions.



Gravity and inertia are opposing forces acting on the Earth's oceans, creating tidal bulges on opposite sites of the planet. On the "near" side of the Earth (the side facing the moon), the gravitational force of the moon pulls the ocean's waters toward it, creating one bulge. On the far side of the Earth, inertial forces dominate, creating a second bulge.

Identify the specific misconceptions in the picture and the text.

12. This picture is commonly seen in elementary textbooks. It shows the lunar gravitational force large on the side of earth nearest the moon, smaller at the earth center, and even smaller on the side opposite the moon. What's misleading about this?



Exam answers.

1. The first picture shows the actual tides being the sum of two tidal bulges, implying that those bulges have independent origins. We have shown this is not so. The second picture speaks of "rotational force", which may mean centrifugal force, but we can't be sure. We also have no clue whether "gravity" means the moon's gravitational attraction, the earth's gravity, or both together.

2. The tidal bulges in this static situation would be essentially the same size as those we have now in mid-ocean. Of course, they wouldn't move across the earth's surface, so the complications due to oceans sloshing around within their shorelines would be absent.

3. A 90° lag would put the moon near the horizon at high tide. The tidal bulge lags the moon by only 3°, so if this were so at shorelines, the tides would arrive late by about 24(3/360)60 = 12 minutes. However, coastal and resonance effects modify this greatly, and there are places where the tides are highest when the moon is at the horizon, but this is not typical. Blackwood uses the word "always", which is clearly inappropriate.

4. "The center of revolution" is ambiguous. It is not one point. Each point on earth revolves around *its own* center of revolution. Only the center of the earth revolves around the barycenter.

5. Arons: "The tide-generating effects now have the same magnitude and the same symmetry as in the existing situation." This is only approximately true, and ignores some small differences due to divergence and gradient of the fields. It's useful to think of this using the superposition principle. A moon of half size produces half as much tidal force. Two such moons 180° apart restore the original situation, approximately. Where the present tides on opposite sides of the earth are slightly unequal, the tides due to two opposing half-size moons would be of equal size on opposite sides of the earth.

6. Materials differ in elastic modulus. Water levels are affected by tractive forces (the horizontal component of the tidal force), which physically moves water into the tidal bulges.

7. Textbooks don't tell you this, do they? The high tide level in water is usually measured from low tide. Coastal tide levels are measured with respect to solid land (not shifting sand) on the shore.

8. There would be no tides in a uniform field. A field gradient is required for a tide.

9. Yes. The elastic modulus of steel and water are different, so this would alter the water pressure as water and steel respond differently to tidal forces. Follow-up question: Would the water pressure inside be higher at high tide, or lower?

10. The ratio of the tidal forces is $(59/61)^3$ 0.90, so they differ by 10%. Compare the difference in gravitational force across the diameter of the earth: $(59/61)^2 = 0.94$, or

about a 6% difference.

11. The picture suggests that the near bulge is only due to gravitation, the other one only due to "inertial forces". The text speaks of "inertial forces", without saying that such a term has no meaning except in a non-inertial coordinate system. The phrase "pulls the ocean waters toward it" implies "motion toward it". The moon exerts gravitational forces on the far side bulge not much smaller than on the near side, and if these forces are "pulling" toward the moon on the near side, they are also pulling toward the moon on the far side. No mention is made of that.

12. The three arrows show gravitational forces due to the moon. No other forces are shown. This leaves the impression that these are the only forces responsible for the tides. But, as we have shown, earth tides are due to the combination of gravitational force due to the moon, gravitational force due to the earth, and tensile forces in the material body of the earth.

a. If the forces shown in the diagram were the only forces acting, then the points A, B, and C would have different accelerations (by Newton's $\boldsymbol{F} = m\boldsymbol{a}$), and the earth would soon be torn apart.

b. Does the picture represent how things are in an inertial frame? If so, then obviously, in view of the above observation, these can't be the only forces acting on the earth. So where are the other forces in the diagram, and what is their source?

c. Does this represent how things are in a non-inertial frame, perhaps rotating about the earth/moon barycenter? If so, then the centrifugal and Coriolis forces should be explicitly shown, for they must be included when doing problems in such a frame of reference?

Gravitational forces due to the moon, gravitational forces due to the earth, and tensile forces of materials are the only real forces acting on the material of the earth. These alone account for the tides.

So that raises the question in the student mind: what accounts for the motion of the earth around the earth/moon barycenter. The answer is simple: the **net** force due to the moon on the body of the earth is solely responsible for that. (We are here ignoring the sun.) It must be so, for (aside from the sun) the moon's gravitational force is the only external force acting on the earth. As students learn in freshman physics, internal forces cannot affect the motion of the body as a whole, for they add to zero in action/reaction pairs. Therefore they need not be included in the equation of motion of the body itself.

I think what irks me about textbook treatments of tides is that they undo the good work we try to accomplish in introductory physics courses. We emphasize correct applications of Newton's laws of motions. First we tell the students to identify the body in question, the body to which we will apply Newton's law. We stress that they must identify the forces on the body in question and only the real forces, due to bodies external to the body in question. We ask students to draw a "free-body" vector diagram showing all these forces that act on the body in question. One must not include forces acting on other bodies. Then sum these forces, to apply $\boldsymbol{F} = \boldsymbol{ma}$. If the net force on the body is non-zero, then it must accelerate. This analysis, done in an inertial system, is adequate to understand the tidal forces, in fact that's the way Newton did it when he discussed tides.

As you notice, these questions were designed deliberately to expose misconceptions arising from misleading textbook and website treatments.

Additional reading.

1. We have treated only the case of tides on a spherically symmetric earth, either an earth with no continents (covered with water), or a solid earth with no oceans. Once you include oceans and continents, resonance effects occur in ocean basins. This can be complex. An excellent treatment of this can be found in Dr. Eugene Butikov's paper "A dynamical picture of the oceanic tides" Am. J. Phys., v. 70, No 10 (October 2002) pp. 1001 – 1011. A pdf version is available online. Dr. Butikov also has by a set of Java-applets) that are beautiful dynamical illustrations of the tide-generating forces and for the wave with two bulges that these forces produce in the ocean.
2. Many textbooks mention that some places on earth, at some times, experience only one tide per day, but few take the trouble to explain why. Steve Kluge has a good explanation of this.
3. Tides and centrifugal force by Paolo Sirtoli. This document has some very nice anaimations that make it all very clear.
4. Tidal Forces and their Effects in the Solar System by Richard McDonald. The larger picture.

Footnote

* The photo of the double lighthouse is a fake. There's only one lighthouse at Folly Beach. However, in keeping with the spirit of this document, these lighthouses ought to be named "Centripetal" and "Centrifugal". [Photo © 2002 by Donald E. Simanek.]

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